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Padé expansion and the renormalization of nucleon-nucleon scattering

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The importance of imposing physical boundary conditions on the T-matrix to remove the nonperturbative renormalization prescription dependence is stressed and demonstrated in two diagonal channels, $^1P_1$ and $^1D_2$, with the help of Padé expansion.

Weinberg's seminal works[1] marked the advent of the effective field theory (EFT) methods in the studies of nucleon systems[2]. However, the nonperturbative character complicates the renormalization of such EFT, which has led to many publications discussing this topic, see, e.g., Refs.[3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. The consensus now arrived is that, for such EFT's, the power counting rules are strongly intertwined with the regularization and renormalization prescriptions. Therefore, a number of power counting schemes have been proposed[7, 9, 10, 11, 12].

On the other hand, the strong interplay between the power counting and renormalization makes the T-matrix develop a nontrivial prescription dependence[13], unlike the perturbative cases[14]. Such phenomenon is already known in other nonperturbative problems[15]. Then the crucial task is to pin down the appropriate counter terms[6, 10], the cutoff scales[4], or whatever parameters[12] in the renormalized T-matrix in a fashion that the physical boundary conditions, like phase shifts, are fulfilled.

In this short report, we wish to further demonstrate and stress our point[13] in the diagonal channels, $^1P_1$ and $^1D_2$. We will employ the Padé expansion of a compact parametrization of the T-matrix proposed before[16] to make our main points about prescription dependence relatively transparent and simple.

The basic framework for describing the nucleon-nucleon (NN) scattering processes at low energies is the T-matrix that satisfies the Lippmann-Schwinger (LS) equation in partial wave formalism,

$$T_{ll'}(p',p;E) = V_{ll'}(p',p) + \sum_{l''}\int \frac{kdk^2}{(2\pi)^2} V_{ll''}(p',k)G_0(k;E^+)T_{l''l'}(k,p;E),$$
$$G_0(k;E^+) \equiv \frac{1}{E^+ - k^2/(2\mu)}, \quad E^+ \equiv E + i\epsilon, \tag{1}$$

with E and μ being respectively the c.m. energy and the reduced mass, $\mathbf{p}$ ($\mathbf{p}'$) being the momentum vector for the incoming (outgoing) nucleon, and $p' = |\mathbf{p}'|$, $p = |\mathbf{p}|$. The potential $V(p',p)$ can be systematically constructed from the χPT according to Weinberg's proposal[**?**]. We remind that the constructed potential is understood to be finite first, as the 'tree' vertices in the usual field theory terminology.

To see the nonperturbative feature, we could transform the above LS equation into the following compact form as a nonperturbative parametrization of T-matrix (we drop all the subscripts for simpleness)[16]

$$T^{-1} = V^{-1} - \mathcal{G}, \tag{2}$$
$$\mathcal{G} \equiv V^{-1}\left\{\int \frac{kdk^2}{(2\pi)^2} V G_0 T\right\} T^{-1}. \tag{3}$$

Here V, T and therefore $\mathcal{G}$ are momentum-dependent matrices in angular momentum space (to show $TT^{-1} = T^{-1}T = I$, we note that,

$$\begin{aligned} TT^{-1} = &\quad (V + V\otimes G\otimes T)T^{-1} = VT^{-1} + V\mathcal{G} = V(T^{-1}+\mathcal{G}) = VV^{-1} = I, \\ T^{-1}T = &\quad V^{-1}T - \mathcal{G}T = V^{-1}T - V^{-1}V\otimes G\otimes T = V^{-1}(T - V\otimes G\otimes T) = V^{-1}V = I, \end{aligned}$$

with $\otimes$ denoting the convolution operation). Unfolding the $\mathcal{G}$ factor, one could readily see that it is an intrinsic nonperturbative factor. The renormalization of the T-matrix is to render this factor free of divergences.

In Ref.[16], it was argued that the perturbative subtraction programme simply fails to renormalize the nonperturbative T-matrix. In Ref.[13], this point was further illustrated by rigorous solutions based on contact potentials, where

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the key conceptual issues were clarified. For example, for contact potentials, the T-matrix for ${}^1S_0$ channel would take the following compact and hence nonperturbative form,

$$\frac{1}{T^{{}^1S_0}(p)} = \frac{\sum_i N_i^{{}^1S_0} p^{2i}}{\sum_j D_j^{{}^1S_0} p^{2j}} + i\frac{\mu p}{2\pi},\ p = \sqrt{2\mu E}, \tag{4}$$

where the parameters $[N_i^{{}^1S_0}, D_j^{{}^1S_0}]$ are functionally dependent upon the EFT couplings (appearing in the potential) and the renormalization prescription parameters (counter terms, cutoffs, subtraction points, etc.).

Thus, to successfully renormalize the nonperturbative $\mathcal{G}$ or T-matrix, the counter terms must be introduced in such a fashion that each of the parameters $[N_i^{{}^1S_0}, D_j^{{}^1S_0}]$ is finite. In general cases, it means that the counter terms must be introduced *before* the corresponding infinite perturbative series are summed up or *before* the corresponding Schrödinger equation is solved[10]. Such counter terms has been termed to be 'endogenous'[13, 16] to stress the difference from the perturbative cases.

Before removing the divergences, the $\mathcal{G}$ factor in a diagonal channel would essentially take the following form

$$\mathrm{Re}(\mathcal{G}_l(p)) = \frac{\sum N_i^{(Bare)}(\Lambda, [C_k])p^{2i} + \text{finite pieces(nonlocal)}}{\sum D_j^{(Bare)}(\Lambda, [C_k])p^{2j} + \text{finite pieces(nonlocal)}}, \tag{5}$$

with Λ being a cutoff scale and $[C_k]$ being the EFT couplings. This is because the divergences brought about with the potential constructed from χPT are mainly power divergences (contained in $[N_i^{(Bare)}, D_j^{(Bare)}]$). In the relatively low energy regions for the NN scattering, say $E \in (0, 200\text{MeV})$, such power like terms dominates the $\mathcal{G}$ factor. Therefore, after subtracting the divergences through endogenous counter terms, the nontrivial prescription dependence in the $\mathcal{G}$ factor (and hence in the T-matrix) is universally parametrized in terms the renormalized constants $[N_i^{(r)}, D_j^{(r)}]$.

This discussion points towards a simple technical treatment of the general parametrization of the nonperturbative prescription of the T-matrix: Parametrize $\mathcal{G}$ in the low energy regions via Padé (or Taylor, in even lower regions) expansion:

$$\mathrm{Re}(\mathcal{G}_l(p))|_{\text{Padé}} = \frac{n_{l;0} + n_{l;1}p^2 + \cdots}{d_{l;0} + d_{l;1}p^2 + \cdots}, \tag{6}$$

$$\mathrm{Re}(\mathcal{G}_l(p))|_{\text{Taylor}} = g_{l;0} + g_{l;1}p^2 + \cdots. \tag{7}$$

Here the coefficients $[n_{l;i}, d_{l;j}]$ or $[g_{l;n}]$ will inevitably be renormalization prescription dependent. Interestingly, such treatment leads to a general parametrization of the renormalization prescription. Now it is straightforward to see the utility of this simple treatment in the following aspects: 1) Much labor is spared in solving the LS equation with various approximation methods and the subsequent renormalization of the T-matrix, which often proves difficult; 2) One avoids being stuck in a specific renormalization prescription when there is a need to examine the prescription sensitivity of the conclusions obtained (This point has been overlooked and caused some disputes on some important issues[17].); 3) One could test whether EFT systematically works for NN scattering in a simple manner. In fact more virtues could be enumerated.

Now our main point becomes obvious: Since the T-matrix must yield the physical predictions, like phase shifts, while different constants of $[n_{l;i}, d_{l;j}]$ or $[g_{l;n}]$ or different prescriptions will give different predictions for the phase shifts, only one set of values for these constants (up to equivalence) could correspond to physical situation. In fact, even if one works with a rational power counting system, the predictions could not be relevant to physical situation *if* the renormalization prescription in use is not fixed by physical boundary conditions. One might doubt that the coarse treatment described about would be useful in practice even if our viewpoint is correct. In the following, to show the efficiency of this seemingly coarse treatment, we demonstrate the phase shift predicted by using such Padé expansion for two diagonal channels, ${}^1P_1$ and ${}^1D_2$. For the ${}^1S_0$ channel case, please see Ref.[18].

The strategy is as follows: 1) First, we choose the truncation of the potential and Padé expansion (or Taylor expansion); 2) Second, we fit to the phase shift data in the low energy ends, say $E(= \mathrm{T}_{\text{lab}}$ in the figures$) \in (0, 10\text{MeV})$, to determine the coefficients (which represent prescription parametrization) in the expansion; 3) Third, the phase shift curves in remaining regions, say, $E \in (10\text{MeV}, 200\text{MeV})$, are predictions. Obviously, the second step is crucial, and corresponds to the step of imposing physical boundary conditions to fix counter terms, cutoff scales or their equivalents in conventional approaches[4, 6, 7, 10, 11, 12].

Following the standard practice[4, 6, 7, 10, 11, 12], we use the PWA data[19] as our targets. Surprisingly, for some diagonal channels, e.g., ${}^1P_1$ and ${}^1D_2$, it suffices to use the simplest expansion: $\mathrm{Re}(\mathcal{G}_l) \approx g_{l;0}$. The results for the

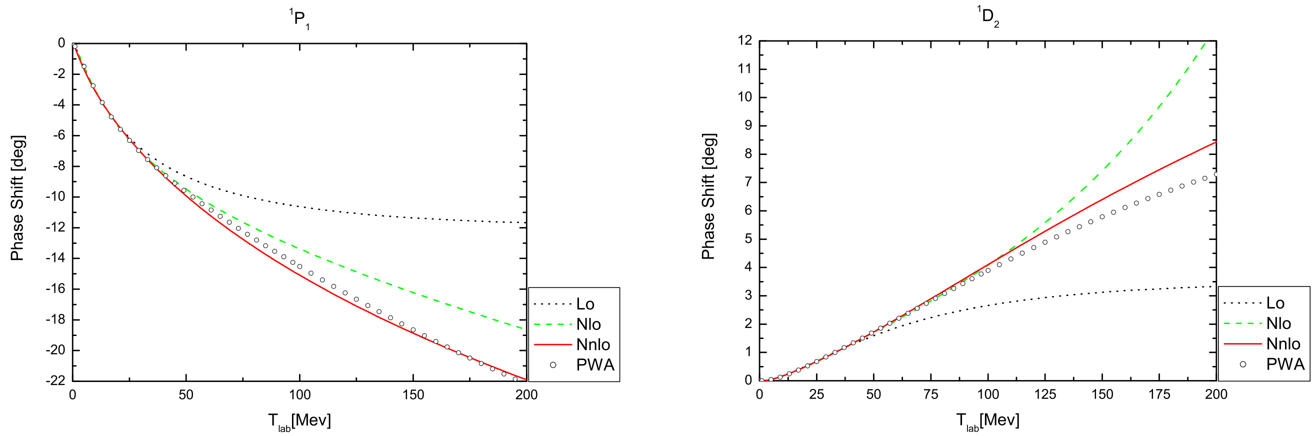

FIG. 1: The predictions of the phase shifts (versus energy $T_{\rm lab}$) for $^1P_1$ channel (left) and $^1D_2$ channel (right) at LO, Nlo and Nnlo, against the PWA data. $\mathrm{Re}(\mathcal{G}_l) = g_{l;0}$ is fitted at lower energy ends

potential truncated at the first three chiral orders (leading order(LO), next-to-leading order (NLO) and next-to-next-to-leading order (NNLO), respectively) are presented in Fig.1. The potential at different orders could be obtained from EGM[4].

It is clear that the predictions of the phase shifts, after fitting out $\mathrm{Re}(\mathcal{G}_{l=1,2}) = g_{l=1,2;0}$, improve significantly as the chiral order for the constructed potential increases (LO, NLO and NNLO). Compared to the results obtained by the conventional approaches[3, 4], our results using the simple treatment (coarse as it seems) are fairly satisfactory, implying that this seemingly coarse approach contains substantial physical contents of the renormalized nucleon-nucleon behavior in the low energy regions. In other words, the efficiency of Padé parametrization of the $\mathrm{Re}(\mathcal{G})$ factor to study the nonperturbative renormalization of the nucleon-nucleon scattering within the EFT framework is essentially justified, Padé expansion of $\mathrm{Re}(\mathcal{G})$ does capture the essences of the nonperturbative feature of T-matrix. The next steps are to apply the strategy based on the Padé expansion of the $\mathrm{Re}(\mathcal{G})$ factor in other channels (including the more interesting coupled channels) and other issues. These works are in progress.

In summary, we have discussed the nonperturbative renormalization of the NN scattering in a compact parametrization of the T-matrix. The main points are explicated in a simple treatment of the $\mathcal{G}$ factor of this compact parametrization based on the Padé expansion. The efficiency of this seemingly coarse but simple approach was demonstrated in two diagonal channels, $^1P_1$ and $^1D_2$.

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